BARRIERS IN COMMUNICATION WHEN SOLVING WORD PROBLEMS

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1. Introduction

When forming mathematical concepts as well as during regular mathematics teaching, the problem of understanding is one of the crucial ones: understanding in the sense of basic communication as well as in the sense of mathematical communication and understanding the subject matter in the whole range of mathematical expressions. It has been shown that in practice, ninety per cent of children's problems in mathematics are caused by problems in communication between the child and the surrounding world. The ability to communicate can be innate or learned. It is the task of the teacher to find out about the specifics of communication of each child and to use them for the teaching of mathematics in the best possible way.

When teaching mathematics, the following types of communication are especially important: communication involved in reading a mathematical text, verbal communication, graphic communication, communication in symbolic pictures and communication in plastic pictures.

In solving word problems and application tasks, we encounter situations in which the child solves simple tasks involving the basic operations with natural numbers without problems, but has difficulties when solving complex word problems. From the wide range of word problems, we have chosen the tasks characterised by the relations "n more (less)" and "n times more (les)". When solving these types of word problems, we encounter problems when the child is unable to perform the analysis of the task and a correct mathematization. A further problem is posed by communication barriers, stemming from not understanding the text and misleading graphic representation and the incorrect solving of the problem as a result of that. We encounter problems in the area of verbal, pictorial, and symbolic communication. In the following, we present the most common problems with solving these tasks.

2. Communication in reading the mathematical text

Reading the word problem task and transcribing it into the mathematical language is a problem for many children. Children often have difficulties with reading the whole text, understanding it, and with coping with the length of the text. They are often unable to understand the question in connection with the task and often answer a different question, which was not even stated in the text and sometimes even does have any connection with the task. Some children have problems with understanding the terminology used (e.g. quarter, takings), some with the expression of relationships through prepositions. They e.g. do not understand the meaning of the prepositional expression "for ... each" in this task: *We bought 8 yoghurts for eight crowns each.* The greatest problem then consists in transcribing this text into mathematical language, i.e. making notes for the calculation, forming the equation, etc.

There are problems with the reading of the symbolic notation and the vision of the child, e.g. the expression 3+5=8 can be understood by the child to mean: three and five are eight, three plus five is eight, when I add five to three, I obtain eight, eight is greater than five by three, eight is greater than three by five, etc.

3. Verbal communication

In order to communicate properly in mathematics, children need to understand the mathematical notions, terms, and relations. This, however, means that the children must have formed a clear image of each of the notions in the sense of its correct definition, even when we do not ask them to memorize the definitions. During verbal communication, both teachers and pupils should focus on essential phenomena, on the facts that are important for the given notion or topic, they should minimize mentioning the properties that are less important and should characterize the notion in a succinct way. It is an advanced skill to be able to express the idea in one's own words and keep the meaning of the notion.

When developing verbal communication, we should monitor whether the children have enough time for expressing themselves verbally in mathematics classes, whether they understand the explanations uttered by the teacher, whether they see and perceive what their teacher expects them to perceive, what their vocabulary is, and how they understand the notions used. Their verbal expression, even when not formulated correctly or in the best way, should not be refused. Children often claim to "not understand it", but they are not able to formulate what the "it" is.

4. Graphic communication

Cultivation of written presentation is the most important means of graphic communication. This concerns especially mathematical notes (e.g. writing of the numbers, notation for algorithms, written operation, brief notes about the task, the process of solving them, as well as the answer) and the layout of the written notes and calculations, which is a prerequisite for correct calculations. Children have difficulties with keeping the same size of the digits, writing on the lines, with correct writing of the numbers in the algorithm schemes, with writing fractions, algebraic expressions, etc.

It is, however, necessary to keep in mind that the good appearance of the layout of the calculations in itself does not guarantee understanding and mastering the topic. It is often the case that children copy the teacher's perfect layout from the blackboard, but do not understand at all what they are writing.

5. Communication in plastic pictures and in symbolic pictures

In communication in plastic pictures, children use the pictures to model mathematical notions and relations. With the help of pictures, we can make word problem tasks and suggestions for ways of solving them and so on more accessible to the children.

Modelling the mathematical situations through a symbolic picture, e.g. symbolic depiction of a word problem or a construction problem in geometry through a simple schematic picture enables the weak pupils to find the solution, but it also makes it easier for the clever ones. It is important that the symbolic depiction is without mistakes and that it expresses a real situation in the task (e.g. the task for addition is depicted differently from the task of comparison using the relation "several more").

Another examples of an illustration between number data are e.g. diagrams used in statistics. Symbolic depiction of numbers in diagrams is much more legible than reading e.g. numbers in tables. There is a general saying that one suitable picture is better than a thousand words.

6. Problems in communication in plastic pictures when solving word problems

The process of solving word problems supposes that the pupil reads the task and understands it and finds out what the core of the question is. Approaches to the solution should be free of any formalism, so that the pupils have as much space for solving the problem as possible. If the pupils have their own insights into the task and solve it without formal notation, we accept their solution. However, since we are not teaching the pupils to solve the individual tasks, but also methods of work, so that they will later be able to solve also more complex tasks, we discuss their ways of solving with them and gradually lead them towards writing the solution down systematically.

There are some principles that should not be overlooked by the pupils. These include especially the analysis of the task, where the pupils clarify for themselves the relationship between the given facts and the ones sought. Graphical representation may be part of this process, because such a representation usually clarifies the task to the pupils. From the picture, writing down the problem mathematically follows (mathematization), then the formal solution of the mathematical task, interpretation of the results of the mathematical task in reality (usually through the answer) and the false-true test.

We have encountered the greatest problems in graphical communication with different types of tasks that are solved by the same calculation, but whose graphic representation is different:

Example 1: Simple word problems

Notice the graphical representation of the tasks:

a) Patrick had 13 beads, he won 5 beads. How many beads did he have after the game?

ooooooooooooo   ooooo   writing down the calculation: 13 + 5 = 18

b) Patrick had 13 beads, Tom had 5 beads more than Patrick. How many beads did Tom have?

P   ooooooooooooo

T   ooooooooooooo   ooooo   writing down the calculation: 13 + 5 = 18

Both tasks, a) and b), are solved with the same calculation, but the reality behind it is different.

The pupils might use graphic representation of the following type:

P   T

ooooooooooooo   ooooo

However, in this picture, it is not clear which beads belong to whom.

c) Patrick had 18 beads at the beginning of the game, but he lost 5 beads. How many beads did he have after the game?

ooooooooooooo ~~ooooo~~ writing down the calculation: 18 - 5 = 13

d) Patrick had 18 beads, Tom had 5 beads less than Patrick. How many beads did Tom have?

P ooooooooooooooooooo

T ooooooooooooo writing down the calculation: 18 – 5 = 13

Similarly as in the example c), the written calculation is the same, but the real situation is different.

A wrong representation

P ooooooooooooooooooo

T ooooooooooooo~~ooooo~~

does not correspond to a comparison task, but to another one, e.g.

Patrick had 18 beads, Tom had 18 beads as well, but he lost 5 beads.

The biggest problem when depicting such tasks lies in the fact that the pupils often depict only the calculation (addition, subtraction), but without a link to the real situation described in the word problem. Representation of word problems characterized by relations of comparison on the number line is not suitable either, since we cannot represent two different objects on one number line.

e) Mum bought 4 exercise books for each of her children. How many exercise books did she buy?

A B C

□□□□ □□□□ □□□□ writing down the calculation: 3 x 4 = 12

A B C

f) Roman won 4 beads and Peter won three times as many beads as Roman. How many beads did Peter win?

R o o o o
P o o o o o o o o o o o o 3times as many

writting down the calculation: 3 x 4 = 12

Wrong representation:

R

P oooo oooo oooo

The picture again does not correspond to the situation in reality.

We also encountered the following representation for this task:

R o o o o
P o o o o o o o o o o o o o o o o plus 3 times as many

g) Grandma had 12 candies and gave it out to three children evenly. How many candies did each child receive?

When solving this example with graphical representation, we give candies one by one to all the children until we run out of them:

A B C

o o o

o o o

o o o

o o o writing down the calculation: 12:3 = 4

h) Philip had 12 candies, Chistopher had three times less candies than Philip. How many candies did Philip have?

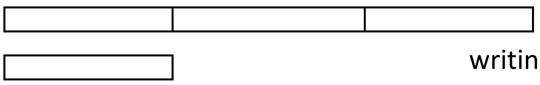
F oooooooooooo

K ? Representing Christopher´s candies is a problem.

It is more advantageous to represent the number of Philip´s candies by a line segment or a rectangle, on which we easily mark the three segments.

F

K



writing down the calculation: 12 : 3 = 4

Example 2: Composed word problems

a) Patrick had 13 beads, Tom had 5 more beads than Patrick. How many beads did they have together?

P ooooooooooooo

T ooooooooooooo ooooo writing down the calculation: 13 + (13 + 5) = 31

b) Patrick has 18 beads, Tom has 5 beads less than Patrick. How many beads do they have together?

P oooooooooooooooooo

T ooooooooooooo writing down the calculation: 18 + (18 - 5) = 31

If the pupils represent the task in an unsuitable way (see e.g. above, 1b)), they usually forget to add the first number (i.e. the number of beads that Patrick has – this is a common complaint of teachers).

c) Patrick and Tom together have 31 beads. Tom has 5 beads less than Patrick. How many beads does each of the boys have?

P o o o o o o o o o o o o o o o o o o

T o o o o o o o o o o o o o

writing down the calculation: (31 – 5) : 2 = 13, 13 + 5 = 18

The following word problem illustrates an **incorrect procedure** in solving an analogous task:

d) Mark and Therese together saved 250 crowns. Theresa saved 50 crown more than Lucas. How many crowns did each of them save?

Calculation: 250 : 2 = 125 125 + 50 = 175 125 – 50 = 75

Theresa saved 175 crown and Lucas 75 crowns,

False-true test: 175 + 75 = 250

The pupils do not take into account (when performing the test), that the difference between the amounts saved by Mark and Theresa is not 50 crowns, as stated in the problem, but 100 crowns.

Example 3: Word problems with anti-signal

a) Patrick has 18 beads, which is 5 more than Tom has. How many beads does Tom have?

P ooooo ooooo ooo|oo ooo
T ooooo ooooo ooo writing down the calculation: 18 – 5 = 13

For some children, "more" is a signal for addition and leads to formal and wrong solving of the problem. When solving the problem, it is necessary to keep emphasizing that if Patrick has 5 beads more than Tom, then Tom has 5 beads less than Patrick, and therefore we subtract.

b) Tom has 13 beads, which is 5 beads less than Patrick has. How many beads does Patrick have?

T ooooo ooooo ooo

P ooooo ooooo ooo ooooo writing down the calculation: 18 – 5 = 13

When discussing the problem, we need to use the fact that if Tom has 5 beads less than Patrick, then Patrick has 5 beads more than Tom. Again, we have the signal "less", but we have to use addition.

In word problems using the expressions "n times less" or "n times more", we encounter a similar situation.

c) Peter has 5 cars, which is three times more than Roman has. How many cars does Roman have?

P ooooo

R ooooo ooooo ooooo writing down the calculation: 3 x 5 = 18

If Peter has three times less cars than Roman, then Roman, has three times more cars than Peter, and therefore we use the addition operation.

d) Roman has 15 cars, and that is three times more than Peter has. How many cars does Peter have?

R ooooo

ooooo

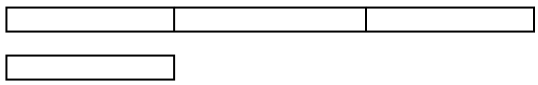
ooooo

T ooooo writing down the calculation: 15 : 3 = 5

If Roman has three times more cars than Peter, then Peter has three times less cars than Roman, and therefore we use division.

e) Together, Roman and Peter have 20 cars. Peter has three times less cars than Roman. How many cars does each of the boys have?

P



T

together: 20

Writing down the calculation: 20 : 4 = 5, 3 x 5 = 15

Peter has 5 cars, Roman has 15 cars.

7. Communication teacher-pupil in education of future teachers

An important part of teacher education consists in bringing to students´ awareness the need for correct communication with pupils during solving word problems. The observations stated below are based on our experiences from observing students – future teachers at elementary school - during their practical internship. They show that some students - future teachers, it is often difficult to choose correct procedure, to ask the pupils suitable auxiliary questions in such a way that solving the word problem would have the desired didactic value.

below, we give an example of a dialogue of a student during setting the task and solving the word problem. The word problem is formulated in such a way that not all the facts necessary to solve the problem are stated in the task. However, these facts can be deduced by solving partial tasks (one or several) for which we do have the necessary information. Here, we have in mind regular composed word problems solved at elementary schools, not “hard nuts to crack”. we have in mind problems that gradually teach the pupils the ways to solve the task, i.e. they are trying to capture the relationship among the facts given, look for necessary information, formulate further questions, etc.

Example of a dialog student S – pupil (pupils) P:

1. S: Read the word problem.

P: Jane wants to buy a DVD which costs 249 CZK. She saves 40 CZK every month. Will she have saved enough in half a year?

2. S: What do we need to do first?

P: Analyse the problem and make notes.

3. S: Do we know everything that we need to know? Do we know how much the DVD costs? How much?

P: 249 CZK

4. S: How much does Jane save?

P: 40 CZK

5. S: For how long has she been saving?

P: Half a year.

6. S: How many months is that?

P: Six.

7. S: So we know that she has been saving for 6 months and that each month, she puts 40 CZK in her money box- How much will she have saved?

P: 6 times 40.

8. S: Calculate: how much is that?

P: 6 times 40 is 240.

9. S: So what do we answer?

P: Jane will have saved 240 CZK,

10. S: What was the question?

P: If she will have saved enough for the DVD.

11. S: So will she have saved enough for the DVD, or not?

P: No, she will lack 9 CZK.

12. S: We must do thetrue-false test. How did we get the 240?

P: 6 times 40 is 240.

13. S: Is that correct?

P: Yes.

From the sample conversation, it is apparent how the dialogue can influence and change the didactic value of the matter taught. By posing the questions that she posed during the dialogue with the children and during solving the task, the student significantly distorted the importance of the individual phases of solving the problem.

Several times during the dialogue, the student posed the question "How many ...?" and the pupils repeated the facts from the word problem formulation. She then herself divided the task into several simple tasks and asked questions that followed one another, and pupils answered those questions easily. By doing that, she significantly diminished the intellectual difficulty of the word problem, because the pupils knew where to look for the information in the formulation by merely finding the corresponding part of the text and reproducing it. The student revealed the way of solving the task, while the purpose of such tasks is that the pupils find out that they do not know everything that is necessary to solve the problem. That information, however, can be found from the formulation of the word problem through further calculations. In other words, it is necessary that the children analyse the task through their own intellectual activity and that they themselves find out that they first need to solve partial simple tasks. By dividing the word problem into several simple tasks herself, the student annulled the effect that solving the task should have brought. Another deficiency in the sample above is the true-false check, which, in fact, has not been performed. Such guidance results in the fact that a number of students cannot later solve word problems independently and on their own. For many a child (and not only a child), word problems are a nightmare. From such a pupil, we usually hear "I do not understand", "I do not know what to do" and without really trying to analyse and solve the task, they often formally and without understanding perform some calculations with the number given.

8. Conclusion

Communication barriers in mathematics can be overcome by choosing suitable procedures and exercises, during which we first try, through individual approach to the pupils, to uncover the communication channels and possibilities of every child and consequently use them to help the child be successful in mathematics.

Form most of the communication barriers given above, we can find remedial teaching exercises that will help children overcome their communication problems. The remedial teaching exercises, however, have to be supported by the child´s own manipulative work, by teaching through experiences, and not by memorizing. It is also necessary to care for the mathematical correctness and precision of the suggested procedures, because e.g. an incorrect representation increases the child´s distrust in mathematics and the deficit in communication can become even greater.

References

GAVORA, P. a kol.: *Pedagogická komunikácia na základnej škole*. Bratislava: Veda, 1988.

HEJNÝ, M., KUŘINA, F.: *Dítě, škola a matematika*. Praha: Portál 2001. 187 s. ISBN 80-7178-561-4.

HOŠPESOVÁ, A. *Školní dialog a řešení slovních úloh*. In MELICHAR J. (ed.) *Matematika v přípravě učitelů elementární školy*. Ústí nad Labem: Univerzita J.E.Purkyně , 2000. s. 170-174,. ISBN 80-7044-301-4.

MAREŠ, J., KŘIVOHLAVÝ, J.: *Komunikace ve škole*. Brno: CDVU 1995. 210 s. ISBN 80-210-1070-3.

MARCHINI, C., KASLOVÁ, M.: *Metody řešení a komunikace*. In: Sborník příspěvků Dva dny s didaktikou matematiky 2003. Praha: PedF UK, 2003.

ROUBÍČEK, F. *Komunikace*. In HOŠPESOVÁ, A., STEHLÍKOVÁ,N., TICHÁ, M. (eds.) *Cesty zdokonalování kultury vyučování matematice.* České Budějovice: Jihočeská univerzita v Českých Budějovicích, 2007. s. 177-205. ISBN 978-80-7394-052-2.